Topology Qualifying Examination

M. Hedden, August 2019

Instructions: Solve four out of the five problems. If you attempt more than four problems, indicate which four you want graded. You must justify your claims either by direct arguments or by referring to theorems you know.

Problem 1. State and prove the Brouwer fixed point theorem.

Problem 2. a) Provide a definition of the term: *deck transformation.*
b) Provide a definition of the term: *normal (or regular) cover.*
c) Give an example of a covering space which is **NOT** normal. Explain why your example isn't normal.

Problem 3. Recall the following definition: A path-connected space whose fundamental group is isomorphic to G and which has contractible universal cover is called a $K(G, 1)$ space.
a) Give an example of a $K(G, 1)$ space (for any group G you wish).
b) Show that any continuous map from a locally path-connected $K(\mathbb{Z}/2, 1)$ space to a $K(\mathbb{Z}/3, 1)$ space is homotopic to a constant map.

Problem 4. Let Σ_g denote the closed orientable surface of genus g. Compute the homology groups of Σ_g.

Problem 5. a) Construct a CW complex whose fundamental group has the following presentation

$$\langle x, y, z \mid x^2yz = 1, y^3 = 1\rangle.$$

Justify your answer.
b) Prove that any group can be realized as the fundamental group of a CW complex.